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Invariants of finite group schemes

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Abstract

Let G be a finite group scheme operating on an algebraic variety X, both defined over an algebraically closed field k. The paper first investigates the properties of the quotient morphism X → X/G over the open subset of X consisting of points whose stabilizers have maximal index in G. Given a G-linearized coherent sheaf on X, it describes similarly an open subset of X over which the invariants in the sheaf behave nicely in some way. The points in X with linearly reductive stabilizers are characterized in representation theoretic terms. It is shown that the set of such points is nonempty if and only if the field of rational functions k(X) is an injective G-modulc. Applications of these results to the invariants of a restricted Lie algebra g operating on the function ring k[X] by derivations are considered in the final section. Furthermore, conditions are found ensuring that the ring k[X]g is generated over the subring of pth powers in k[X], where p = chark > 0, by a given system of invariant functions and is a locally complete intersection.

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